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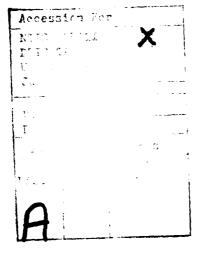
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# 20. ABSTRACT CONTINUED

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"Applications of Bound and Mode Approximation Techniques for Large Deflections of Pulse Loaded Structures"

FINAL REPORT

bу

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21 December 1981

U.S. Army Research Office Grant Number DAAG29-78-G-0085

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## 1. Problem Studied

The problem studied in our research has been how to estimate the response of structures of ductile metal to high intensity pressure pulse loading or to impact, in conditions where permanent ("plastic") deformations occur. The aim has been to develop reliable approximate methods of analysis which would be simpler and cheaper than wholly numerical (finite element or finite difference) approaches. The latter are available through a variety of codes, but the dedirability of simpler methods to supplement them needs no argument. It is worth re-emphasizing, however, that a main advantage of a sound approximation technique, over a wholly numerical one, is its aid in understanding the basic mechanics of the response, free of unessential detail and irrelevant complications.

# 2. Subjects Treated

The difficulties in treatments of transient structural response either analytically or numerically have stemmed largely from the complex mixture of material behaviors: elastic, plastic and viscoplastic. The latter must be considered because important structural metals, especially steel, have inelastic behavior strongly sensitive to rates of deformation, and these vary enormously over the structure and during the response. In our research we have therefore tried to achieve a large degree of simplification by artificially separating the response into distinct elastic and plastic (or viscoplastic) stages. Strain rate dependence is included by iterative determination of a dynamic yield stress. Our work has been concerned with implementing this three-stage approach to the problem, and with better individual treatment of the separate stages.

The "three-stage" approach consists of (i) an initial wholly elastic response; (ii) a subsequent wholly plastic response; and (iii) a final elastic vibration. The initial stage is determined by the load pulse, and may be carried out using either a computer program for elastic transient analysis or a simplified model of finite component type. The "wholly plastic" second stage which follows is a solution assuming rigid-perfectly plastic behavior. In the simplest type of analysis this is taken in mode form (a full solution of the dynamic plastic equations with fixed shape over the structure, all quantities having the same time variation). Alternatively an initial non-modal form may be used. This is analyzed by methods of rigid-plastic analysis. A basic feature of such behavior is that the initial pattern evolves into a final pattern in mode form. The third stage consists of elastic vibrations starting from rest and governed by linear equations. The other initial condition is that the initial strains corrrespond through Hooke's law to the final stress field of the preceding rigid-plastic mode. The mean amplitude of the residual vibrations is subtracted from the maximum deflections of the previous stage to obtain an estimate of the permanent deflection.

The first elastic stage must provide starting conditions for the ensuing plastic stage in a way which corresponds realistically to the transmission of momentum. Our technique for doing this takes the initial velocity magnitudes of the rigid-plastic motion so as to minimize the mean square difference between these velocities and those at the end of the preceding elastic stage. This is the "minimum  $\Delta_0$ " technique proposed by Martin and Symonds [1] as the optimal means of determining the initial velocity amplitude of an approximate rigid-plastic solution in mode form to a problem of impulsive loading, where

initial velocities are specified. It corresponds to matching momentum integrals over the structure, kinetic energy not, in general, being conserved. (The same method may be applied in an initial velocity problem of an elastic problem, to obtain an approximate solution using a finite number of normal modes. The application of the minimum  $\Delta_0$  method in the elastic problem gives results identical with those of classical methods.)

The formulation of a "global" yield condition, which determines the termination of the initial elastic stage, is the most arbitrary step in the procedure. The basis for the choice is the initial location of plastic hinges in the rigid-plastic phase which is taken to follow the elastic stage, and in work to date the "effective stress" of the yield condition has been taken as either a root-mean-square or an arithmetic mean of absolute values of bending moments at the plastic hinge locations in the ensuing rigid-plastic motion.

## 3. Main Results

Some of the results have been published as indicated in the list of reports. Two reports are in preparation. The titles, publication references and abstracts are given below.

#### Reports

1. "The Optimal Mode in the Mode Approximation Technique" by P.S. Symonds, Mechanics Research Communications Vol. 7(1), pp. 1-6, 1980.

## Abstract

The paper proposes an addendum to a simple technique [1] for obtaining approximate magnitudes of maximum permanent deflections and response times, in a structure subjected to pulse loading. This technique obtains an approximate solution in terms of mode form (separated variable) full solutions of the field equations, with initial amplitude of the velocity field chosen so as to minimize the initial value of a functional  $\Delta(t)$  of the actual response velocity and the approximating mode solution,  $\Delta_{\bullet}^{min}$ representing the greatest difference between the two fields in a squared-integrated sense. Apart from other advantages, the least value of min was suggested as a means of identifying the particular mode, from a family of available mode solutions, which would provide the best approximation. The present note shows by a simple lumped-mass model that this criterion fails if unstable as well as stable modes are considered. A better criterion is here proposed; namely, that the largest value of a lower bound on the root-mean-square final deflection identifies the mode giving the best approximation. This lower bound is derived and illustrated together with other possible criteria.

 "Finite Elastic and Plastic Deformations of Pulse Loaded Structures by an Extended Mode Technique", P.S. Symonds, Int. J. Mech. Sci. Vol. 22, pp. 597-605, 1980.

#### Abstract

A scheme for estimating maximum transient and permanent deflections of a pulse-loaded structure is outlined and applied to fully clamped beams of mild steel, titanium alloy 6Al-4V, and aluminum alloys 7075-T6 and 6061-T651. The method makes use of concepts of the mode approximation technique [1] and extends previously proposed schemes [2,3]. Comparisons are made with deflections measured in tests [4-6] as well as with predictions from rigid-plastic mode analyses and from numerical solutions.

3. "Elastic-Plastic Deflections due to Pulse Loading", P.S. Symonds, Reprinted from the Proceedings of the Specialty Conference on Dynamic Response of Structures, ASCE/Atlanta, GA/January 15-16, 1981

#### Abstract

The paper describes and illustrates the application of an approach to estimating deflections (peak transient and permanent) or structures subjected to pulse loading. Mode form rigid-plastic solutions furnish the main plastic response. These are introduced and succeeded by elastic motions, so that "first order" influences of elastic effects are included.

4. "Approximate Elastic-Plastic Deflections of a Portal Frame due to Pulse Loading on One Column", J.R. Raphanel and P.S. Symonds, ARO 79/4 (in preparation)

#### Abstract

A three-stage approximate solution is written in which an initial elastic solution is succeeded by a rigid-plastic response which absorbs the remaining kinetic energy, and which in turn is followed by residual elastic vibrations. The mean amplitudes of these are taken to approximate the permanent deflection field of the structure. The rigid-plastic stage includes both a first phase with changing shape of response pattern, and a final stage in mode form. Hence, both "local" deformations (in the loaded column) and general sidesway deflections are estimated. Some comparisons are made with numerical solutions by a finite element program.

5. "Convergence onto Mode Form Solutions in Impulsively Loaded Rigid-Plastic Structures", J.B. Martin, ARO 79/5, (in preparation)

#### Abstract

The paper considers models in load and velocity space which illustrate the behavior of impulsively loaded piecewise linear rigid-plastic structures. Attention is focussed on the question of determining onto which mode a particular initial velocity field will converge, or conversely, what initial velocity fields converge onto a particular mode. Some answers to this question

are provided, and links with the determination of the optimal time bound and with a recently presented algorithm for determining mode shapes are established.

Items (2) - (4) are concerned with the simplified elastic-plastic approach outlined above. Items (1) and (5) are concerned with dynamic rigid-plastic analysis, in particular with questions concerning the mode shape that the impulsively loaded structure will converge on, and the choice of mode shape to be used for best results in an approximation technique.

These two questions are basic ones in dynamic plastic structural analysis, whether or not elastic effects are important.

## 4. Further Results

Much work has been done under the grant that will have future use, but has not yet been fully reported, concerning computer programs for transient structural response. Such programs are of interest in connection with both the elastic-plastic approach and the study of properties of rigid-plastic response. Two general computer programs were made available to us by outside sources.

The first such program was HONDO, a finite element program for structures with axial symmetry and quite general elastic-plastic material behavior. The tape for a version with viscoplastic behavior (HONDO II) was kindly provided in September 1978 by its author, Dr. S.W. Key, of Sandia Laboratories. Dr. C. Dittmer intended to use this to examine applications of approximate methods to impulsively loaded circular plates with fully restrained edges and strongly nonlinear behavior at finite deflections. An extended version of the mode approximation technique made use of "instantaneous mode" solutions appropriate to current deflection fields, and including the transition from fluxural to membrane action [2,3]. Despite Dr. Dittmer's experience in computing, many difficulties were encountered in implementing the new version of HONDO on our IBM machine, and no fully satisfactory "exact" solutions were obtained with it. He did make many calculations of interest in connection with the large deflection plate problem, writing his own programs for various modifications of the mode technique. These have not yet resulted in a report because of the absence of exact numerical solutions with which to compare them, and because results similar to his had then been presented by another investigator [4]. Comparisons with experimental results have been made [3,5] but exact solutions by computer are also desirable in this problem because of experimental difficulties in providing the full edge constraint condition [6].

The second program made available to us was ABAQUS, a general purpose program whose original version was written by Dr. H.D. Hibbitt. In September 1980 it was provided free for our research purposes by Hibbitt and Karlsson, Inc., of Providence, RI, and implemented on the VAX 11/780 mini-computer recently acquired for the solid mechanics group. This is a large program with many advanced features, including an automatic time step control based on a computed force residual at mid-interval. It had not previously been run on a VAX machine, and its implementation required considerable time spent by our staff, particularly Dr. Uee-Wan Cho. It has been running successfully during much of the past year. So far it has mainly provided the equivalent of "test" points for comparisons with predictions of the approximate methods. However, an objective

tive of present work is to use such a program for the elastic response in the simplified elastic-plastic approaches described in Reports 2-4, and in other publications [7,8].

Further work as yet unreported was done by Dr. H. Kim, on the problem of determining mode shapes in rigid-plastic structure loaded impulsively, and undergoing finite deflections so that geometric nonlinearities need to be included. The "instantaneous mode" approach referred to above requires the determination of mode form solutions at fixed deflection fields (including the initial state). Kim did this by a Rayleigh-Ritz type of procedure based on a minimum principle for velocity fields: a mode form solution renders the plastic dissipation rate a minimum, for all fields with the same kinetic energy. Kim devised an effective iteration scheme for this minimization based on the DFP method ("Davidson, Fletcher, and Powell"). He had applied this method to a fully fixed beam [9], and attempted to apply it to the plate with fully constrained edge. Here he met with further computational difficulties that he was not able to resolve in the available time. He has since worked independently on this problem, and a paper may eventually result, in which full acknowledgement of ARO support will of course be given.

It should be mentioned that recently a new and better method has been obtained by Martin [10] for the determination of mode shapes. Martin's method involves minimizing the same dissipation functional that Kim was using. However Kim's direct approach involves minimization subject to a quadratic constraint condition (constant kinetic energy). Martin showed that this can be replaced by a sequence of linear minimization problems. The key idea is that if a trial velocity field is held constant, minimization of the dissipation rate subject to this linear constraint condition leads to a new velocity field for which the basic functional is smaller than for the trial velocity field. The new velocity field can in turn be taken as the trial velocity field, and held constant in a second minimization of the dissipation. There is a valuable analogy with static limit analysis. Each such linear minimization can be shown to be exactly analogous to the minimization which leads to an upper bound on the collapse load, the constant load pattern of the static problem being analogous to the momentum field of the trial velocity field in the dynamic problem.

The new method is apparently extremely efficient. Report (5) provides further elaboration of this important contribution to basic theory of dynamic rigid-plastic response, which in turn plays a key role in the general approximation scheme including elastic effects.

As a result of the present ARO grant considerable progress has been made both in basic theory and in practical methods for estimating plastic deformation due to pulse loading. The further work needed is clearly indicated. There is a primary need for further experimental work; comparisons with numerical solutions, while helpful, cannot be regarded as validation for real structures. In addition, there is a need for putting the whole procedure in the most accessible form for new users. It seems probable that rather simple but fairly general programs suitable for micro-computers can be written, and this is an objective of future work.

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# 5. Scientific Personnel Participating

Professor P. S. Symonds (Principal Investigator)
Professor J. B. Martin (Visitor)
Dr. C. T. Dittmer (Research Associate)
Dr. H. Kim (Research Associate)
Dr. Uee W. Cho (Research Associate)

J. R. Raphanel (Research Assistant/Graduate Student) (requirements for Ph.D. completed September 1981)

